

Evaluate the following limits

1 $\lim_{x \rightarrow 3} \frac{x^3 + x^2 - 9x - 9}{x - 3}$

2 $\lim_{x \rightarrow 2} \frac{x^2 + 4x - 12}{x^2 - 2x}$

3 $\lim_{x \rightarrow 0} (\sin x)^{\sin x}$

4 $\lim_{x \rightarrow \infty} \arcsin(\cos(\arctan(5x)))$

5 $\lim_{x \rightarrow 1^-} \frac{d}{dx} 2|x - 1|$

6 $\lim_{x \rightarrow -\infty} \frac{20x^2 - 19x + 18}{17 + 16x - 15x^2}$

Compute the derivative.

① $1 + 2 \sin x + 3 \cos(4x)$

② $e^x \cos(3x)$

③ $(\arctan(2x - 2))^2$

④ $\frac{x}{x^2+5} + 4x^3$

⑤ $(\ln |x|)^{x^3}$

Find all possible antiderivatives

① $\cos(5x + 10)$

② 4^x

③ $x^{\frac{3}{4}}$

④ $\ln |10x|$

⑤ $\frac{x}{x^2+5} + 4x^3$

⑥ $2xe^{x^2}$

Bob needs Alice's help! Alice is 100 feet South of the edge of a 100 foot wide river. Bob is on the opposite bank 150 feet East of the point on the river bank nearest to Alice. If Alice is 4 times faster on land than she is in water, what is the fastest route to Bob?

Give the equation of the tangent line through the point $(1, 2)$ and tangent to the ellipse defined by the equation $x^2 - 2x + 4y^2 - 8y = -1$

The function $f(x) = 10 - 3x + x^2 - \frac{1}{3}x^3$ is invertible. Find the value of the derivative of the inverse function $f^{-1}(x)$ at $x = 189$.
HINT: $f(9) = 189$.

Use the Squeeze Theorem of Limits to evaluate the limit $\lim_{x \rightarrow 0} x^2 \cos(\sin(x))$. HINT: Use that $-1 \leq \cos u \leq 1$ for any u .

What are the results of the first three iterations if Newton's Method is applied to approximate a root of the polynomial $x^4 - x^3 - x - 1$ nearest to 2?

The derivative of the function f is given by $f'(x) = (x - 1)^2(x - 5)(x - 10)$. Identify the local maxima and local minima.

Find all the critical points of the function $h(x) = \min\{|x + 1|, |x - 3|\}$. Decide if each point is a local minimum, local maximum, or neither. Is h continuous?

Find all the inflection points of the function

$f(x) = \frac{1}{42}x^7 + \frac{2}{15}x^6 - \frac{3}{20}x^5 - \frac{3}{2}x^4 + 12x - 1$. Determine the intervals on which f is concave up. Determine the intervals on which f is concave down.

Find the area between the curve $y = \ln |3x|$ and the x axis and between the vertical lines $x = 1$ and $x = 5$.

Let f be a function on the interval $[0, 5]$ defined by

$$f(x) = \begin{cases} \frac{1}{x-3} & \text{if } 0 \leq x < 1 \text{ or } 4 < x \leq 5 \\ -x^2 + 4x + 1 & \text{if } 1 \leq x \leq 4 \end{cases}$$

Find all the critical points of f and determine if each is a local max, min, or neither. Does f achieve an absolute maximum? minimum?

At what time is the function $g(t) = -2 \arctan(5t - 100)$ decreasing the fastest?

You have 10 cm of string to make a square and a regular hexagon. What is the largest possible product of the square and hexagon areas?

Evaluate $\log_{10}(\cos(\arctan(3)))$ sans calculator.

Alice and Bob start out 200 miles apart. Alice drives toward Bob's initial location at 40 mph, and at the same time Bob starts driving away going 25 mph and perpendicular to Alice's motion. After 2 hours of driving at what rate is the distance between the two cars changing? Is it increasing or decreasing?

I need a box shaped building 3 times as long as it is wide with a volume of 1000 cm^3 . The vertical material for the box costs twice what the horizontal material will cost. Find the dimensions of the cheapest box that meets the criteria.

The function $f(x) = \frac{x^3}{1+e^x}$ has a unique global maximum. Use Newton's Method to approximate the x value where it is achieved. Initialize with $x_0 = 3$ and compute the first two iterations.

Identify the inflection points, local maxima, and local minima of the function $g(x) = 2x - \sin(4x)$ on the interval $[-5\pi/12, 5\pi/12]$. What is the absolute maximum and absolute minimum?