1. The table below contains some values of a function $f : \mathbb{R} \to \mathbb{R}$ evaluated at different values of x.

x	$f(x)$
-2	12
3	10
4	11
5	6
7	7

(a) (3 points) What is the average rate of change of f on the interval $[-2, 4]$?

$$\frac{f(4)-f(-2)}{4-(-2)} = \frac{23}{6}$$

(b) (3 points) Give the definition of the instantaneous rate of change of f at the point $x = 1$.

$$\lim_{\Delta x \to 0} \frac{f(1+\Delta x) - f(1)}{\Delta x}$$

2. (a) (3 points) Find the limit

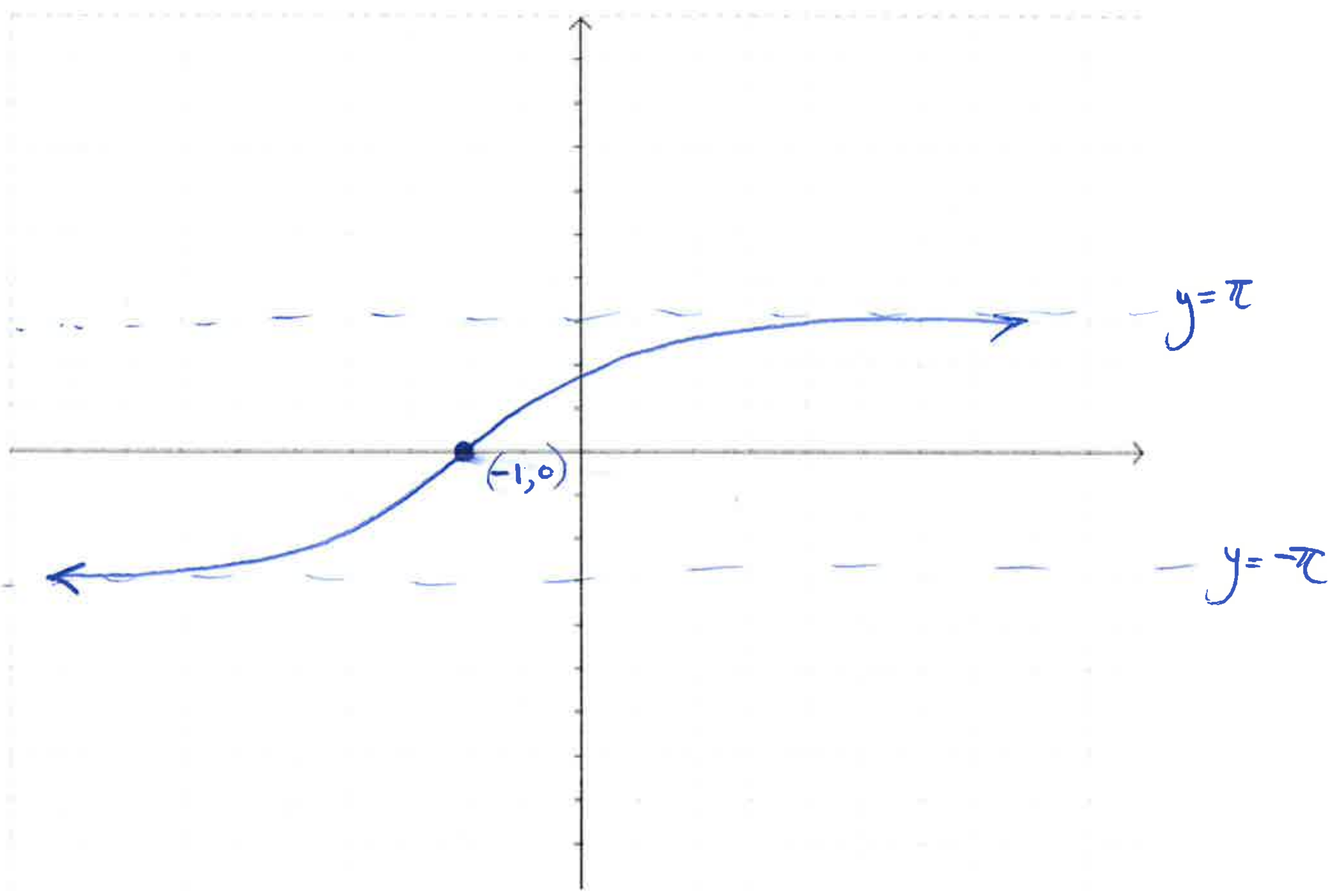
$$\lim_{x\to-\infty}\left(\frac{1-x^3}{x^2+3x^3-42}\right)^3$$

$$=\left(\lim_{x\to-\infty}\frac{1-x^3}{x^2+3x^3-42}\right)^3$$

$$=\left(\frac{-1}{3}\right)^3$$

$$=\frac{-1}{27}$$

(b) (3 points) Sketch the graph of the function $2\arctan(x+1)$. Indicate the values at which asymptotes and zeros occur.



3. Consider the functions $g(t) = \frac{1}{\sqrt{25-t^2}}$ and $h(t) = \sqrt{t+4}$

(a) (3 points) What is the domain of g?

Domain of $g =$ $(-5, 5)$

(b) (3 points) Is g invertible? Why or why not?

No. g is not one-to-one.

$g(1) = g(-1)$

(c) (3 points) Give the composition function $g \circ h$ evaluated at t.

$g \circ h(t) = \dfrac{1}{\sqrt{25 - \sqrt{t+4}^{\,2}}} = \dfrac{1}{\sqrt{21-t}}$ for $-4 \le t < 21$

(d) (3 points) Compute the limit.

$\lim_{t \to 5^-} \log_5 |\sqrt{5-t}| + \log_5 (g(t))$

$$\lim_{t \to 5^-} \log_5 \left| \frac{\sqrt{5-t}}{\sqrt{25-t^2}} \right|$$

$$= \lim_{t \to 5^-} \log_5 \left| \frac{1}{\sqrt{5+t}} \right|$$

$$= \log_5 \left| \frac{1}{\sqrt{10}} \right| = -\frac{1}{2} \log_5 10$$

4. Consider the piecewise defined real function g defined on all real numbers but -1.

$$g(x) = \begin{cases} -1 & \text{if } x \leq -2 \\ \frac{x^2-1}{x+1} & \text{if } -2 < x < -1 \text{ or } -1 < x \leq 0 \\ \frac{1}{\cos\left(\frac{1}{x}\right)} & \text{if } x > 0 \end{cases}$$

(a) (3 points) Give a real number j where g has a jump singularity.

$j = -2$

(b) (2 points) Give a real number a where g has an oscillatory singularity.

$a = 0$

(c) (2 points) Give a real number p where g has a pole/divergent singularity.

$p = \frac{2}{\pi}$

$$g(x) = \begin{cases} -1 & \text{if } x \le -2 \\ \frac{x^2-1}{x+1} & \text{if } -2 < x < -1 \text{ or } -1 < x \le 0 \\ \frac{1}{\cos\left(\frac{1}{x}\right)} & \text{if } x > 0 \end{cases}$$

(d) (3 points) Give a real number r where g has a removable discontinuity. What is the value of $\lim_{x\to r} g(x)$?

$r =$ -1

$\lim_{x\to r} g(x) =$ $\lim_{x\to -1} \frac{x^2-1}{x+1} = \lim_{x\to -1} x-1 = -2$

(e) (3 points) Compute the limit

$\lim_{x\to+\infty} g(x) =$ $\lim_{x\to\infty} \frac{1}{\cos(\frac{1}{x})} = \lim_{u\to 0^+} \frac{1}{\cos(u)} = 1$

(f) (3 points) Compute the limit

$\lim_{x\to-2^+} g(x) =$ $\lim_{x\to -2^+} \frac{x^2-1}{x+1} = \lim_{x\to -2^+} x-1 = -3$

5. (a) (3 points) State the Intermediate Value Theorem.

Let f be continuous on the interval $[a,b]$. Then for any value y between $f(a)$ and $f(b)$ there is some $c \in (a,b)$ such that $f(c) = y$.

(b) (3 points) Explain how you know there must be a solution to the equation

$$7^x + x^2 = 5$$

for some $x \in (0,1)$.

Let $f(x) = 7^x + x^2$. f is continuous. $f(0) = 1$ and $f(1) = 8$ so by the intermediate value theorem there is some $c \in (0,1)$ such that

$$f(c) = 7^c + c^2 = 5.$$

(c) BONUS: (3 points) Suppose that $f : \mathbb{R} \to \mathbb{R}$ is an odd function and $\lim_{x \to 0+} f(x) = 10$. What can you conclude?

$$\lim_{x\to 0^-} f(x) = \lim_{u \to 0^+} f(-u) = -\lim_{u \to 0^+} f(u) = -10$$

So there is a jump singularity of f at 0.