Section 2.1 : Rates of Change and Tangents to Curves

Chapter 2 : Limits and Continuity

Math 1551, Differential Calculus

"All truths are easy to understand once they are discovered; the point is to discover them." - Galileo Galilei

Section 2.1 Rates of Change and Tangents to Curves

Topics

1. Free fall.
2. Estimating the rate of change of a function.
3. The equation of a tangent line to a function at a point.

Learning Objectives

For the topics in this section, students are expected to be able to:

1. Characterize motion of a falling object using Galileo's law to estimate distance travelled and speed.
2. Estimate and compute the rate of change of a function.
3. Construct the equation of a tangent line to a function at a point.

Free Fall

- Galileo discovered that: an object falling from rest, under the force of gravity, falls a distance d proportional to the time that it has been falling, t.

$$d(t) = 16t^2\ ,\ t \geq 0$$

- This type of motion is referred to as **free fall**.

Example 1

An object undergoing free fall. Estimate the speed of the object:

a) over the interval from 1 to 2 seconds

b) over the interval from 1 to $1 + \Delta t$ seconds, $\Delta t > 0$

Average Rate of Change

Definition

The **average rate of change** of $y = f(x)$, with respect to x, over interval $[x_1, x_2]$ is

$$\frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(x_1 + h) - f(x_1)}{h}$$

Note:

- $\Delta x = x_2 - x_1 = h \neq 0$.
- The line passing through points $(x_1, f(x_1))$ and $(x_2, f(x_2))$ is a **secant line**.

In-Class Participation Activity: Worksheet

(if time permits)

The remainder of the examples in this lecture are incorporated into a worksheet.

- Please solve worksheet problems in groups of 2 or 3 students
- Each group submits **one** completed worksheet
- Clearly print full names at the top of your sheet
- Every student in a group gets the same grade
- Grading scheme per question:
  - 0 marks for no work, for students working by themselves, or for working in a group of 4 or more
  - 1 mark for starting the problem or for a final answer with insufficient justification
  - 2 marks for a complete solution

Secant and Tangent Lines

Example

Suppose $y(x) = 4 - x^2$. Construct the equations of:

a) the secant line that passes through $y(x)$ at $x = 1$ and $x = 1 + h$.

b) the tangent line at $x = 1$.

Also describe what happens to the secant line as $h \to 0$.

Instantaneous Rate of Change

The **instantaneous** rate of change of a function $f(t)$ at $t = t_1$ is given by the slope of the tangent line at that point.

Example

The graph below gives the position of a car, $p(t)$, as a function of time t.

a) Is the car moving faster at time t_1 or t_3?

b) Is the car speeding up or slowing down at times t_1 and t_2?

c) What happened between times t_4 and t_5?