Section 2.2 : Limit of a Function and Limit Laws

Chapter 2 : Limits and Continuity

Math 1551, Differential Calculus

Section 2.2 Limit of a Function and Limit Laws

Topics

We will cover these topics in this section.

1. Limits of Functions
2. The Sandwich (or Squeeze) Theorem

Learning Objectives

For the topics in this section, students are expected to be able to:

1. Determine whether limits exist, where they exist, and if they do, evaluate them.
2. Evaluate limits using the Sandwich Theorem.

Limit

What value does $f(x)$ approach as x approaches 3?

$$f(x) = \frac{x^2 - 9}{x - 3}$$

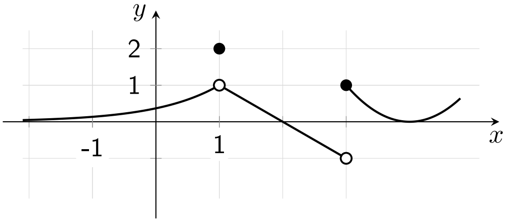
The Concept of a Limit

- The limit of $f(x)$ as x approaches a is the value of f as we

- We denote the limit of $f(x)$ as x approaches a as

Example 1

The graph of a function, $y(x)$, is shown below.



Determine the value of the following limits.

$$\lim_{x\to 1} y(x)$$

$$\lim_{x\to 3} y(x)$$

Existence

- In order for the limit to **exist**, we must approach the **same** value from the ___________ and ___________ of the limit point.
- **Right hand limit**: the number that f approaches as x "nears" a from the right $(x > a)$.

- **Left hand limit**: a number that f approaches as x nears a from the left $(x < a)$.

If the limit of $f(x)$ as $x \to a$ exists, then:

Limit Theorems

Suppose

$$\lim_{x \to a} f(x) = L, \qquad \lim_{x \to a} g(x) = M, \qquad c \in \mathbb{R}$$

Then

$$\lim_{x \to a} (f(x) \pm g(x)) =$$

$$\lim_{x \to a} (f(x)g(x)) =$$

$$\lim_{x \to a} cf(x) =$$

$$\lim_{x \to a} \frac{f(x)}{g(x)} =$$

Rational Functions and Polynomials

If $P(t)$ is a polynomial, then $\lim_{t\to a} P(t) =$

If $R(t) = \dfrac{P(t)}{Q(t)}$ is a rational function, then

$$\lim_{t\to a} R(t) = \lim_{t\to a} \frac{P(t)}{Q(t)} =$$

But what if $Q(a) = 0$?

Sandwich Theorem

Suppose

$$g(x) \leq f(x) \leq h(x)$$

for all x in some open interval containing c, except possibly at $x = c$. Suppose also that

$$\lim_{x \to c} g(x) = \lim_{x \to c} h(x) = L$$

Then:

Examples (as time permits)

Evaluate the limits, if possible.

1. $\lim\limits_{t\to 0} f(t)$, where $1 - t^2 \le f(t) \le 1 + t^2$.
2. $\lim\limits_{t\to 0} \dfrac{1}{t^2}$
3. $\lim\limits_{t\to 0} \dfrac{\sqrt{1+t} - \sqrt{1-t}}{t}$