# Section 3.2 : The Derivative as a Function

Chapter 3 : Differentiation

Math 1551, Differential Calculus

*"Solving a problem for which you know there's an answer is like climbing a mountain with a guide, along a trail someone else has laid. In mathematics, the truth is somewhere out there in a place no one knows, beyond all the beaten paths. And its not always at the top of the mountain. It might be in a crack on the smoothest cliff or somewhere deep in the valley."*

- Yōko Ogawa

# Section 3.2 The Derivative as a Function

## Topics

1. The derivative of a function
2. Sketching the derivative of a function.
3. Differentiability

## Learning Objectives

For the topics in this section, students are expected to be able to:

1. Compute the derivative of a function
2. Construct the equation of a tangent line at a point.
3. Sketch the derivative of a function over an interval, or sketch a function given a graph of its derivative.
4. Identify where functions are differentiable.

# $f'(x)$

**Definition**

The **derivative** of $f(x)$ at $x$ is itself a function, and is defined as

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h},$$

provided that the limit exists.

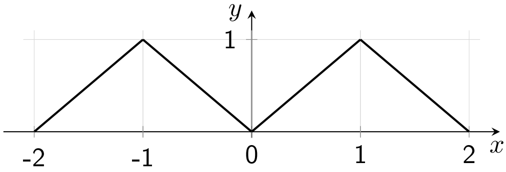
If the above limit exists at every point in the domain of $f$, then we say that $f$ is **differentiable**.

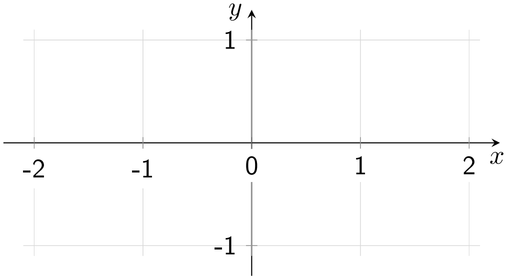
Common notations for the derivative include:

$$y'(x) = \frac{dy}{dx} = \frac{d}{dx}\left(y(x)\right)$$

# Example

The graph of $y(x)$ on $x \in [-2, 2]$ is shown below. Sketch $y'(x)$.



# Differentiability

**Definition**

A function $f(x)$ is **differentiable on an open interval** $(a, b)$ if its derivative exists everywhere on that interval.

$f(x)$ is **differentiable on a closed interval** $[a, b]$ if it is differentiable on $(a, b)$ and these limits exist:

$$\lim_{h \to 0^+} \frac{f(a+h) - f(a)}{h} \qquad \lim_{h \to 0^-} \frac{f(b+h) - f(b)}{h}$$

# Differentiability and Continuity

**True or False**

a) If a function is differentiable at a point, then it is also continuous at that point.

b) If a function is continuous at a point, then it is also differentiable at that point.

# Additional Example (if time permits)

Suppose $y(x) = \frac{1}{x+2}$.

a) Sketch $y(x)$.

b) Construct the equation of the tangent line at $x = -1$.

c) Draw the tangent line on your graph.

# Key Points

- The derivative of $f(x)$ at $x$ is itself a function
- $f(x)$ is not differentiable at $x_0$ if any of the following are true:
  - $x_0$ is not in the domain of $f(x)$
  - $f(x)$ has a discontinuity at $x_0$
  - $f(x)$ has a vertical tangent at $x_0$
  - The graph of $f(x)$ has a sharp point at $x_0$