Section 3.4 : The Derivative as a Rate of Change

Chapter 3 : Differentiation

Math 1551, Differential Calculus

"Math is the language of the universe. So the more equations you know, the more you can converse with the cosmos."

– Neil deGrasse Tyson (@neiltyson) November 21, 2011

Section 3.4 The Derivative as a Rate of Change

Topics

1. Velocity, speed, acceleration.

Learning Objectives

For the topics in this section, students are expected to be able to:

1. Compute the velocity, speed, and acceleration of a moving object, given its position as a function of time.
2. Give examples of expressions and draw graphs that represent the motion of a moving object.
3. Interpret equations and graphs that represent the motion of a moving object.

Note that the textbook also explores marginal costs, jerk, and sensitivity to change, which we won't have time to cover and students are not expected to know.

Instantaneous Rate of Change

Recall:

- We can compute derivative of $f(t)$ at t_0 using

$$f'(t_0) = \lim_{\Delta t \to 0} \frac{f(t_0 + \Delta t) - f(t_0)}{\Delta t}$$

- $f'(t_0)$ can be positive, negative, or zero.
- $f'(t_0)$ represents:
    - the rate of change of $f(t)$ at t_0
    - the slope of the tangent line of $f(t)$ at t_0

Velocity and Speed

Suppose the function $s(t)$ describes the position of a moving object.

Definition

The **displacement** of the object over time interval $t + \Delta t$ is

$$\Delta s = s(t + \Delta t) - s(t)$$

The **average velocity** is

$$v_{ave}(t) = \frac{\Delta s}{\Delta t}$$

The **instantaneous velocity** is

$$v(t) = \frac{ds}{dt}$$

Speed is the absolute value of velocity:

$$\text{speed} = |v(t)|$$

Acceleration

Definition

Acceleration is the derivative of $v(t)$ with respect to time:

$$a(t) = \frac{dv}{dt} = \frac{d^2 s}{dt^2}$$

Position, Velocity, and Acceleration Can be Negative

Badwater Basin, CA, is 282 feet below sea level, and is the point of lowest elevation in the US.

Image by J. Couperus, www.flickr.com/photos/jitze1942

If $h(t)$ is the height above sea level, then, as you are walking down a hill into the basin:

Speed is Non-Negative

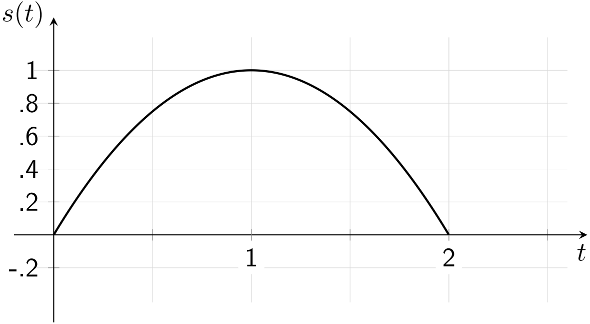
Note that speed can't be negative.

(for example: speedometers in cars don't have negative numbers)

Image by J. Backlund, www.flickr.com/photos/joelbacklund

Example 1

The graph below gives the position of a moving object, $s(t)$, for $t \in [0, 2]$.



Estimate the times when:

a) the speed of the object is equal to zero

b) the velocity is negative

Example 2

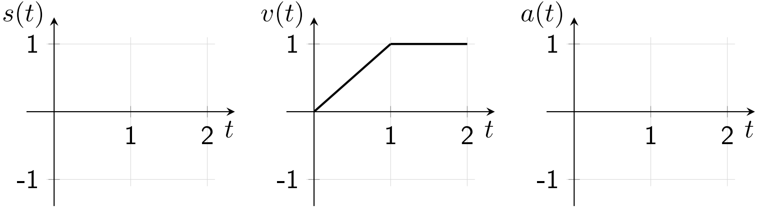
A moving object at time t has negative velocity for $t \in [0, 4]$, and when $t = 2$ its acceleration is zero.

a) Give a formula that could represent the objects' position for $t \in [0, 4]$.

b) Sketch a graph that could represent the objects' position for $t \in [0, 4]$.

Example 3

The velocity of a moving object, $v(t)$, for $t \in [0, 2]$ is shown below. Sketch the position $s(t)$, and the acceleration $a(t)$. Assume $s(0) = -1$.



Example 4 (if time permits)

The height of an object is given by $h(t) = 2 + 16t - 32t^2$.

a) Use a derivative to determine when the object reaches its maximum height.

b) What is the maximum height?