Section 4.3 : Monotonic Functions, the First Derivative Test

Chapter 4 : Applications of Derivatives

Math 1551, Differential Calculus

Section 4.3 Monotonic Functions, the First Derivative Test

Topics

1. Identifying where functions are increasing and where they are decreasing.
2. The first derivative test.

Learning Objectives

For the topics in this section, students are expected to be able to:

1. Determine where a function is increasing or decreasing.
2. Classify critical points using the first derivative test.
3. Sketch functions using the first derivative and the first derivative test.

Motivation

- In sketching the graph of a function it is useful to know where it increases and where it decreases over an interval.
- This section gives a test to determine where a function increases and where it decreases.
- We also explore one method for testing the critical points of a function to identify whether local extreme values are present.

Increasing and Decreasing Functions

Definition

Let f be a function defined on domain D, x_1 and x_2 are two points in D, and $x_2 > x_1$.

- If $f(x_2) > f(x_1)$, then f is **increasing** on D.
- If $f(x_2) < f(x_1)$, then f is **decreasing** on D.

Example

Give a formula for an even function $f(x)$ that is increasing for $x \in (-\infty, 0]$, and decreasing for $x \in [0, \infty)$.

Participation Activity: Index Card

- Please work in groups of two or three
- Each group submits **one** completed card
- Print full names at the top of your card
- Every student in a group gets the same grade
- Grading scheme per question:
  - 0 marks for no work or for students working by themselves
  - 1 mark for starting the problem or for a final answer with insufficient justification
  - 2 marks for a complete solution
- Print today's date at the top, which is ____________

The activity consists of one or two of the examples in this lecture. Your instructor will pass out index cards.

Example 1

If possible, give a formula for a continuous function, $f(x)$, that satisfies the following criteria. If it is not possible to do so, state why.

a) Domain D is $[0, 2]$, f is increasing on D, $f'(x) < 0$ on D.
b) Domain D is $[0, 2]$, f is increasing on D, $f'(x) = 0$ on D.
c) Domain D is $[0, 2]$, f is increasing on D, $f'(x) > 0$ on D.
d) Domain D is $[0, 2]$, f is decreasing on D, $f'(x) < 0$ on D.

Derivatives and Increasing and Decreasing Functions

Definitions

Let f be a differentiable function.

- If $f'(x) > 0$ on (a, b), then f is **increasing** on $[a, b]$.
- If $f'(x) < 0$ on (a, b), then f is **decreasing** on $[a, b]$.

A function that is increasing (or decreasing) on an interval is **monotonic** on that interval.

Example 2

Let $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$.

a) Determine where the function is increasing, and where it is decreasing.

b) Identify the local extrema of the function and where they are located.

c) Sketch $f(x)$.

The First Derivative Test

Suppose f has a critical point at $x = c$.

- If $f'(x)$ changes from positive to negative at c, then f has a **local maximum** at c.
- If $f'(x)$ changes from negative to positive at c, then f has a **local minimum** at c.
- If $f'(x)$ doesn't change sign from positive to negative at c, then f has no local minimum or maximum at c.

Additional Examples (if time permits)

1. If possible, sketch a function $f(x)$ that is odd, continuous, $f'(x) < 0$ on $(-1, 0)$, local minimum at $x = 1$.
2. $f(x) = x^2 - x - \ln x$.
   a) Determine where the function is increasing, and where it is decreasing.
   b) Identify the local extrema of the function and where they are located.
   c) Sketch $f(x)$.