Section 4.4 : Concavity and Curve Sketching

Chapter 4 : Applications of Derivatives

Math 1551, Differential Calculus

Section 4.4 Concavity and Curve Sketching

Topics

1. Identifying where functions are concave up and concave down.
2. The second derivative test.
3. Curve sketching.

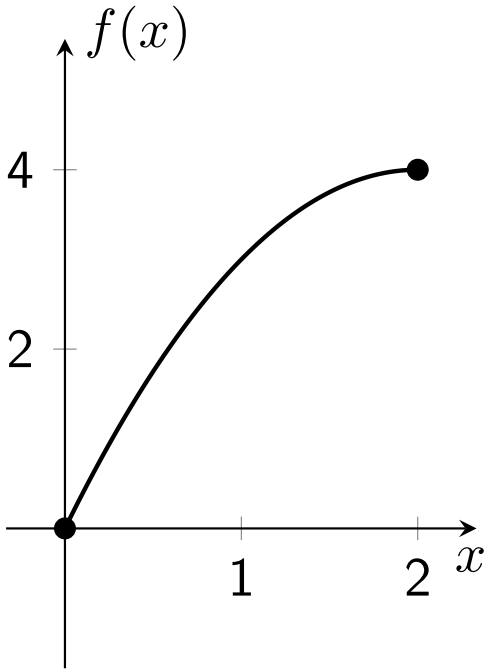
Learning Objectives

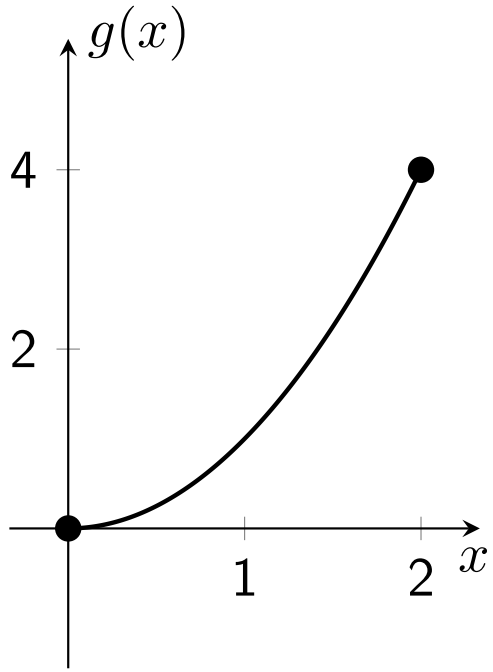
For the topics in this section, students are expected to be able to:

1. Determine where a function is concave up or concave down.
2. Classify critical points using the second derivative test.
3. Sketch functions using characteristics such as concavity, intervals of increasing/decreasing, extrema, symmetry, intercepts, asymptotes, domain and range.

Motivation

Consider the following continuous functions. How are they different from each other?





Concavity and Inflection Points

Definition

The graph of a differentiable function $f(x)$ is

- **concave up** on an open interval if $f''(x) > 0$
- **concave down** on an open interval if $f''(x) < 0$

Definition

An **inflection point** is a point where the graph of f changes concavity.

We can locate inflection points by identifying where $f''(x) = 0$, or where $f''(x)$ DNE.

Example

Identify the intervals of concavity, inflection points, and local extrema of $f = x^4 - 4x^3$. Sketch the curve.

Second Derivative Test

Suppose f has a critical point at $x = c$.

- If $f''(c) > 0$, then f has a local minimum at c.
- If $f''(c) < 0$, then f has a local maximum at c.
- If $f''(c) = 0$, then the second derivative test is inconclusive.

In-Class Participation Activity: Worksheet

Some of the examples in these slides are incorporated into a worksheet.

The usual grading scheme applies:

- Please solve worksheet problems in groups of 1 to 3 students
- Each group submits **one** completed worksheet
- Clearly print full names at the top of your sheet
- Every student in a group gets the same grade
- Grading scheme per question:
  - 0 marks for no work, for students working by themselves, or for working in a group of 4 or more
  - 1 mark for starting the problem or for a final answer with insufficient justification
  - 2 marks for a complete solution
- Print today's date at the top, which is _____

Examples (as time permits)

For the following functions, determine:

i) the domain

ii) all asymptotes

iii) symmetry (even, odd, neither)

iv) critical points, intervals where f is increasing/decreasing

v) inflection points and intervals of concavity

vi) local and absolute extrema

Use the information above to sketch $f(x)$. Label your axes.

a) $f(x) = \dfrac{2x^2}{x^2 - 1}$

b) $f(x) = \ln(4 - x^2)$