Section 4.8 : Antiderivatives

Chapter 4 : Applications of Derivatives

Math 1551, Differential Calculus

"The only real valuable thing is intuition" - Albert Einstein

In this section we will apply the intuition we have developed throughout the course when calculating derivatives, to calculate antiderivatives.

Section 4.8 Antiderivatives

Topics

1. Integration
2. Integration and differential equations

Learning Objectives

For the topics in this section, students are expected to be able to:

1. Construct antiderivatives and indefinite integrals of functions.
2. Apply indefinite integrals to solve differential equations and initial value problems.

Motivation

There are many times when we want to obtain an expression for a function when we are given the first or second derivative of a function.

For example:

- Relationships developed in areas of science, social sciences, and engineering give us equations involving $f'(x)$ or $f''(x)$, from which we need to construct $f(x)$.
- Sometimes it is easier to measure $f'(x)$ or $f''(x)$ than it is to measure $f(x)$.

Question: given $f'(x)$, how can we construct an expression for $f(x)$?

Participation Activity: Worksheet

- Please work **by yourself or with one other person**
- Each group submits **one** completed sheet
- Print full names at the top of your sheet
- Every student in a group gets the same grade
- Grading scheme per question:
  - 0 marks for no work
  - 1 mark for starting the problem or for a final answer with insufficient justification
  - 2 marks for a complete solution
- Print today's date at the top, which is ___________

The activity consists of one or two of the examples in this lecture. Your instructor will pass out worksheets.

Example

Identify a function $f(x)$ whose derivative is given. Check your answer in each case by differentiation.

1. $3x$
2. $5x^2$
3. $5x^2 + 3x$
4. $\sin(x)$
5. $\sec^2(x)$
6. $\cos(2x)$
7. e^{2x}
8. $\sqrt{x}$
9. $\frac{1}{x}$ for $x > 0$
10. 3^x

The Antiderivative

Definition

The function $F(x)$ is an **antiderivative** of $f(x)$ if $F'(x) = f(x)$.

The set of all antiderivatives is defined as the **indefinite integral**

$$\int f(x)dx = F(x) + C$$

C is a constant, $f(x)$ is the **integrand**, $\int$ is an **integral sign**, and dx is a differential that indicates the variable over which we are integrating.

Example

Calculate the indefinite integrals.

a) $\int \left(3t^2 + 1\right) dt$

b) $\int \left(\frac{1}{\sqrt{x}} + 4x^5\right) dx$

Differential Equations

Definition

A **differential equation** is an equation that involves derivatives.
A **solution** to a differential equation is a function that satisfies the differential equation.

Example

Identify a solution to the differential equation $y'(x) = 2x$.

Initial Value Problems

Definition

A differential equation of the form $\frac{dy}{dt} = f(t)$ subject to the condition $y(t_0) = y_0$ is an **initial value problem**.

Example

Solve the initial value problem

$$\frac{dy}{dt} = \frac{4}{\sqrt{t+1}}, \quad y(0) = 0$$

Table of Integrals

function	antiderivative
$x^n,\ n \neq -1$	$\dfrac{x^{n+1}}{n+1}$
$\dfrac{1}{x}$	$\ln \lvert x \rvert$
e^{kx}	$\frac{1}{k} e^{kx}$
b^x	$\frac{1}{\ln b} b^x$

These integrals will not be given on the final exam. Please memorize them.

Table of Integrals

function	antiderivative
$\sin kx$	$-\frac{1}{k}\cos kx$
$\cos kx$	$\frac{1}{k}\sin kx$
$\sec^2 kx$	$\frac{1}{k}\tan kx$
$\csc^2 kx$	$\frac{1}{k}\cot kx$
$\sec kx \tan kx$	$\frac{1}{k}\sec kx$
$\csc kx \cot kx$	$\frac{1}{k}\csc kx$

These integrals will not be given on the final exam. Please memorize them.

Additional Examples (if time permits)

1. True or false.
   a) $\int x \cos x \, dx = x^2 \sin x + C$
   b) $\int \frac{1}{3}(2x+3)^2 \, dx = (2x+3)^3 + C$
   c) $\int \sqrt{2x+1} \, dx = \sqrt{x^2 + x + C}$
2. A sketch of $y'(t)$ is shown on the left. If $y(0) = 0$, sketch $y(t)$.