Final Exam Review Exercise Set C, Math 1551, Fall 2017

As stated in the syllabus, a goal of this course is to prepare students for more advanced courses that have this course as a pre-requisite. To help us meet this goal, **solutions are not are provided for these problems**. This is intentional: upper level courses often don't have solutions to everything. So students need, develop, and use various strategies to check their solutions in those courses. Students are encouraged to ask questions they may have about these problems on Piazza, office hours, by checking their answers with their peers. Calculators and software are also great ways to check your work. All of these methods are valuable skills that are transferable to higher level courses.

1. Evaluate the limits, if they exist.

   (a) $\lim_{x\to 0} \frac{1}{x^2}\sin^2\left(\frac{x}{2}\right)$

   (b) $\lim_{x\to 4} \frac{x-4}{|x-4|}$

   (c) $\lim_{x\to 0} \frac{x}{x-|x|}$

   (d) $\lim_{x\to 5^-} \frac{\sqrt{2x}(x-5)}{|x-5|}$

   (e) $\lim_{x\to -4^+} (x+5)\left(\frac{|x+4|}{x+4}\right)$

   (f) $\lim_{x\to 0^-} \frac{9}{7x^{1/3}}$

   (g) $\lim_{x\to 0} \frac{\sqrt{1+x}-1}{x}$.

   (h) $\lim_{x\to \infty} \frac{2x^3+3x^2}{x-5x^2}$

   (i) $\lim_{x\to \infty} \frac{-9x^2+6x+5}{-13x^2-7x+15}$

   (j) $\lim_{x\to 0} 6x^2(\cot 3x)(\csc 2x)$

   (k) $\lim_{x\to 0} \frac{\sin(\sin x)}{\sin x}$

   (l) $\lim_{x\to 0} \frac{\sin x\cos 4x}{x+x\cos 5x}$

   (m) $\lim_{x\to \infty} (4x-\sqrt{16x^2-6x+9})$

   (n) $\lim_{x\to \infty} \sqrt{\frac{16x^2}{7+9x^2}}$

   (o) $\lim_{h\to 0} \frac{(x+h)^3-x^3}{h}$

2. Identify all asymptotes of

$$f(x)=\frac{2-2x-x^2}{x}$$

3. Determine the point(s) at which $f(x)=\frac{x}{x-2}$ has slope $-1/2$.

4. Graph the rational function $f(x) = x^3/(x^2 - 49)$.

5. At what points is the function continuous?

$$f(x) = \frac{x+3}{x^2 - 12x + 35}.$$

6. Construct the equation of the tangent line to the curve $x^6 + x^3 = y^2 + 12x$ at $(0, 1)$. Then construct the equation of the normal line.

7. Compute the derivative dy/dx for the following.

   (a) $\cos(xy) + x^6 = y^6$

   (b) $y = \ln\left(\dfrac{e^x}{7 + e^x}\right)$

   (c) $y = e^{\cos^2(\pi x - 4)}$

   (d) $y = \cos^2(\cot^6 x)$

   (e) $y = \sqrt{9 + x \sin x}$

   (f) $y = \dfrac{1}{(x^2 - 2)(x^2 + 3x + 4)}$

   (g) $y = x^{1-x}$

   (h) $y = x^{4x}$

8. Calculate the second derivative of $y = \frac{x^4+2}{x^2}$.

9. Water is flowing at a rate of $50 \text{ m}^3/\text{min}$ from a shallow concrete conical reservoir (vertex down) of base radius 45 m and height 6 m. (a) How fast (cm/min) is the water level falling when the water is 5 m deep? (b) How fast is the radius of the water's surface changing then? Answer in cm/min.

10. The edge length x of a cube decreases at the rate of 2 m/min. At what rate does the cube's surface area change when x = 6m? At what rate does the cube's volume change when x=6m?

11. The volume of a rectangular box with a square base remains constant at 500 cm^3 as the area of the base increases at a rate of 9 cm^2/sec. Find the rate at which the height of the box is decreasing when each side of the base is 19 cm long.

12. One airplane is approaching an airport from the north at 170 km/hr. A second airplane approaches from the east at 217 km/hr. Find the rate at which the distance between the planes changes when the southbound plane is 28 km away from the airport and the westbound plane is 23 km from the airport. Round to the nearest whole number.

13. Use differentials to estimate $(8.5)^{1/3}$.

14. Given that $y' = x^{-2/3}(x - 3)$, find y'' and sketch the graph of $y = f(x)$.

15. Sketch the graph of the function $y = \frac{18x}{x^2+9}$.

16. A company is constructing an open-top, square-based, rectangular metal tank that will have a volume of 66 cubic feet. What dimensions yield the minimum surface area?

17. Suppose a business can sell x gadgets for $p(x) = 250 - \frac{x}{100}$ dollars per gadget, and it costs the business $c(x) = 1000 + 25x$ dollars to produce x gadgets. Determine the production level and cost per gadget required to maximize profit.

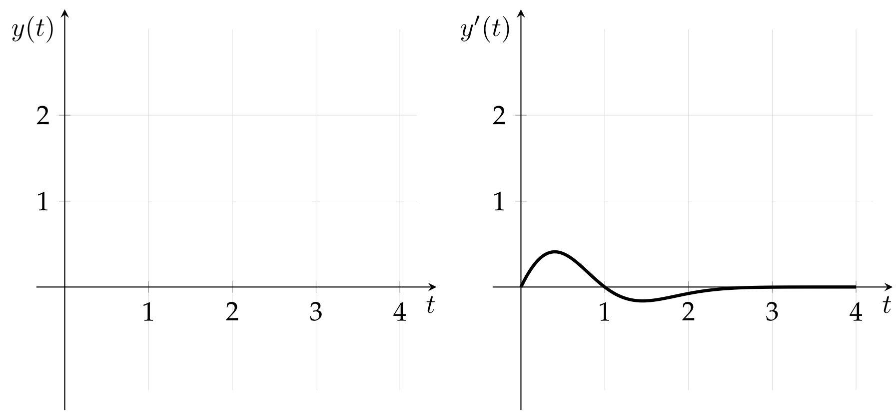
18. Identify the function's local and absolute extreme values, if any, saying where they occur.

$$h(x) = \frac{x-1}{x^2+5x+10}.$$

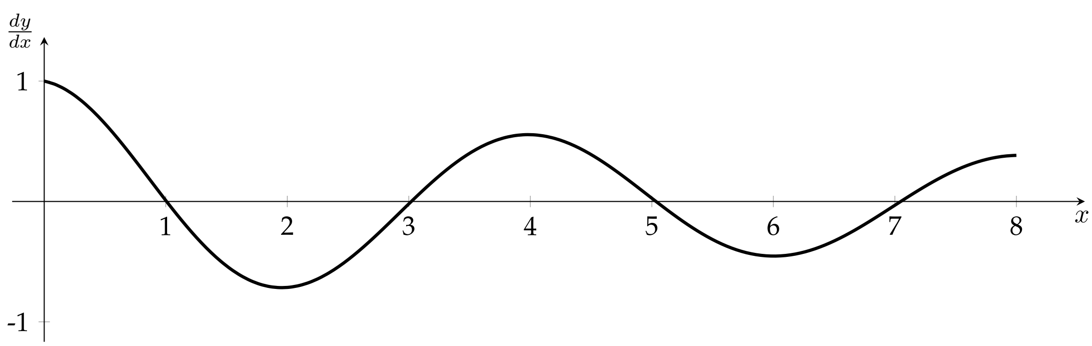
19. Let $f(x) = 3 + 4x$. Use the formal definition of limit to show that $\lim_{x \to 1} f(x) = 7$.

20. Use Newton's Method to estimate a solution to $5x^3 + 9x^2 + x + 1 = 0$. Start with $x_0 = 0$ and calculate x_2.

21. A sketch of $y'(t)$ is shown on the right over the interval $t \in [0, 4]$. If $y(0) = 1$, sketch $y(t)$.



22. The graph below shows the derivative, $\frac{dy}{dx}$, of a function, $y(x)$.



Assume that the domain of y is $[0, 8]$.

(a) On what intervals is y increasing?

(b) For what values of x, if any, does y have a local maximum?

(c) For what values of x, if any, does y have a local minimum?

(d) On what intervals is y concave up?

(e) For what values of x does y have an inflection point?

23. If possible, sketch the graph of a function that satisfies the following criteria. If it is not possible to do so, state why. It isn't necessary to give a formula for the functions. Assume that the function is continuous and differentiable everywhere (unless stated otherwise).

(a) $f(x)$ odd, $f(2) < -1$, and $\lim_{x \to \infty} f(x) = -1$

(b) $g(x)$ is continuous, even, $\lim_{x \to -\infty} g(x) = -2$, and $\lim_{x \to \infty} g(x) = 2$

(c) $y(x)$ is odd and is not differentiable at exactly two points.

(d) $f(x)$ has a local minimum at $x = 0$, and $f'(x) > 0$ over the interval $(-1, 1)$.

(e) $f(x)$ has local maxima at $x = 0$ and $x = 1$, $f(x)$ has no local minima.

(f) $f(x)$ has local maxima at $x = 1$ and $x = 2$, and is concave up for $x > 0$.

24. Solve the initial value problems. You may assume that $t \geq 0$.

(a) $y'(t) = 6\pi \cos(3\pi t) + \dfrac{1}{t+1}, \quad y(0) = 2.$

(b) $y'(t) = e^{-t} + 2t + 3, \quad y(0) = 0.$