Math 1551-M2A **Name:** ____________________
Fall 2017
Exam 1A
22 September 2017
Time Limit: 50 Minutes

This exam contains 6 pages (including this cover page) and 5 questions. There are 43 points in total. Write explanations clearly and in complete thoughts, as if you are explaining the problem to somebody with similar knowledge. No calculators or notes may be used. Put your name on every page.

Grade Table

Question	Points	Score
1	9	
2	10	
3	8	
4	8	
5	8	
Total:	43	

Formal Symbols Crib Sheet

$f : A \to B$	function with domain A & codomain B	$\mathbb{N}$	natural numbers
$f \circ g$	composition of functions	$\mathbb{Z}$	integers
f^{-1}	inverse function	$\mathbb{Q}$	rational numbers
$\lim_{x\to a}$	limit as x approaches a	$\mathbb{R}$	real numbers
$\lim_{x\to a^-}$	limit from below	(a, b)	open interval a to b
$\lim_{x\to a^+}$	limit from above	$[a, b]$	closed interval a to b
$\subset$	subset of	$\in$	element of
$\cap$	intersection	$\cup$	union
$\mapsto$	maps to		

1. (9 points) Compute the values of the following limits or explain why they do not exist.

(a)

$$\lim_{x\to 4} \frac{(\sin(3x-12))^2}{x-4}$$

Remember that $\lim_{x\to 0} \frac{\sin x}{x} = 1$ so $\lim_{x\to 4} \frac{\sin(3x-12)}{(3x-12)} = 1$ by a change of variables. Then the desired limit is $\lim_{x\to 4} \frac{3\sin(3x-12)\sin(3x-12)}{3(x-4)} = 3 \cdot 1 \cdot 0 = 0$

(b)

$$\lim_{x\to\infty} \frac{3x\sin x - 1}{x}$$

Simlify to see the limit is $\lim_{x\to\infty} 3\sin x - \frac{1}{x} = \lim_{x\to\infty} 3\sin x$. So the limit does not exist since sin is a periodic function that will continue to oscillate.

(c)

$$\lim_{x\to -1^+} \frac{x+1}{x+|x+1|+1}$$

Since $x > -1$ we have $|x+1| = x+1$ and we can simplify

$$\lim_{x\to -1^+} \frac{x+1}{x+|x+1|+1} = \lim_{x\to -1^+} \frac{x+1}{2x+2} = \lim_{x\to -1^+} \frac{1}{2} = \frac{1}{2}$$

2. (10 points) (a) What is a function? A mathematical function consists of a set of inputs

called the domain, a set of outputs called the range, and a rule that gives exactly one output for every input.

(b) What does it mean for a function to be continuous?

A function is continuous if for every limit point of its domain, the function value and limit are defined and equal. That is,

$$\lim_{x \to a} f(x) = f(a)$$

for any a approached by the domain.

(c) Find the values of the real numbers A and B so that the piecewise function f defined below is continuous.

$$f(x) = \begin{cases} 2x^3 - 2x + 1 \text{ if } x \leq 0 \\ Ax + B \text{ if } 0 < x < 1 \\ x^4 - x^3 - 6 \text{ if } x \geq 1 \text{ originally misprinted as } 0 \end{cases}$$

To be continuous at 0 the two formulas on the different sides must be equal. so

$$\lim_{x \to 0^-} f(x) = 1 = B = \lim_{x \to 0^+} f(x)$$

and to be continuous at 1

$$\lim_{x \to 1^-} f(x) = A + B = -6 = \lim_{x \to 1^+} f(x)$$

So $B = 1$ and $A = -7$.

3. (8 points) (a) Find a formula for the inverse function f^{-1} of the function

$$f(x) = \sqrt{\log_4\left(\frac{1}{x}\right)}$$

$$f^{-1}(x) = 4^{-x^2}$$

(b) An investment account with \$1000 pays continuously compounded interest at an annual rate of 4%, so that the value $V(t)$ of the account after t years is $V(t) = 1000e^{0.04t}$. How many years will it take for the investment to triple? (Give your result as a functional expression—you do not need to give a decimal number.)

If t_0 is the time when the investment triples we have $e^{0.04t_0} = 3$ so

$$t_0 = \frac{\ln 3}{0.04}$$

4. (8 points) (a) What is an asymptote?

An asymptote for the graph of a function $y = f(x)$ is a line that the graph approaches arbitrary close as x or y tend to $\pm\infty$.

(b) Find all the asymptotes of the following rational function g.

$$g(x) = \frac{2(x-2)^2}{x+2}$$

There is a vertical asymptote at $x = -2$ and since $\frac{2(x-2)^2}{x+2} = 2x - 12 + \frac{32}{x+2}$ an oblique asymptote given by $y =$

5. (8 points) (a) What does the Squeeze Theorem of Limits say?

The Squeeze Theorem says that if there are functions f, g, and h such that $f(x) \leq g(x) \leq h(x)$ for all x near a and if the limits $\lim_{x\to a} f(x) = \lim_{x\to a} h(x)$ exist then so does $\lim_{x\to a} g(x)$ and $\lim_{x\to a} f(x) = \lim_{x\to a} g(x) = \lim_{x\to a} h(x)$

(b) An important function in statistics is the Gauss error function, written $\text{erf}(x)$. Like sin or cos, it has no easy formula. But for any real number x, the Gauss error function does satisfies the inequality

$$\frac{2}{\sqrt{\pi}}\left(1-\frac{x^2}{3}\right) \leq \frac{\text{erf}(x)}{x} \leq \frac{2}{\sqrt{\pi}}\left(1-\frac{x^2}{3}+\frac{x^4}{10}\right)$$

Use the inequality (or any other method) to compute the limit

$$\lim_{x\to 0} \frac{\text{erf}(x)}{x} =$$

According to the Squeeze theorem we can compute the limit of the two bounding functions of x. Setting $x = 0$ in to both of these gives a value of $\frac{2}{\sqrt{\pi}}$ so

$$\lim_{x\to 0} \frac{\text{erf}(x)}{x} = \frac{2}{\sqrt{\pi}}$$