Math 1551-G **Name:** ______________________
Fall 2015
Exam 4
7 December 2015
Time Limit: 170 Minutes

This exam contains 12 pages (including this cover page). Only answer 7 of the first 8 free response questions. Cross off the question you do not want graded from the grade table below. If you do not cross off a question the first 7 will be graded. There are 5 multiple choice bonus problems at the end of the exam. Put your name on every page of the exam.

Honor Pledge: I have read and understand the exam instructions. I commit to uphold the ideals of honor and integrity by refusing to betray the trust bestowed upon me as a member of the Georgia Tech community.

Signature: ____

Grade Table

Question:	1	2	3	4	5	6	7	8	9	Total
Points:	9	9	9	9	9	9	9	9	0	63
Score:										

Formal Symbols Crib Sheet

$f : A \to B$	function with domain A & codomain B	$\mathbb{N}$	natural numbers
$f \circ g$	composition of functions	$\mathbb{Z}$	integers
f^{-1}	inverse function	$\mathbb{Q}$	rational numbers
$\lim_{x\to a}$	limit as x approaches a	$\mathbb{R}$	real numbers
$\lim_{x\to a^-}$	limit from below	(a, b)	open interval a to b
$\lim_{x\to a^+}$	limit from above	$[a, b]$	closed interval a to b
$\subset$	subset of	$\in$	element of
$\cap$	intersection	$\cup$	union
$\mapsto$	maps to	f'	derivative
$\frac{d}{dx}$	derivative with respect to x		

Derivatives Crib Sheet

For constant $a \in \mathbb{R}$ and arbitrary real functions f and g

Function	Derivative	Function	Derivative
a	0	af	af'
$f+g$	$f'+g'$	fg	$f'g+fg'$
$\frac{f}{g}$	$\frac{f'g-fg'}{g^2}$	$f \circ g$	$(f' \circ g)g'$
f^{-1}	$\frac{1}{f' \circ f^{-1}}$	x^a	ax^{a-1}
a^x	$a^x \ln a$	$\log_a \lvert x \rvert$	$\frac{1}{x \ln a}$
$\sin x$	$\cos x$	$\csc x$	$-\csc x \cot x$
$\cos x$	$-\sin x$	$\sec x$	$\sec x \tan x$
$\tan x$	$\sec^2 x$	$\cot x$	$-\csc^2 x$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$	$\mathrm{arccsc} x$	$\frac{-1}{\lvert x \rvert \sqrt{x^2-1}}$
$\arccos x$	$\frac{-1}{\sqrt{1-x^2}}$	$\mathrm{arcsec} x$	$\frac{1}{\lvert x \rvert \sqrt{x^2-1}}$
$\arctan x$	$\frac{1}{1+x^2}$	$\mathrm{arccot} x$	$\frac{-1}{1+x^2}$
$\sinh x$	$\cosh x$	$\cosh x$	$\sinh x$

Geometry Crib Sheet

Pythagorean Identity $a^2+b^2=c^2$

Circle: radius r	$A=\pi r^2$	$c=2\pi r$
Box: dimensions x, y, z	$V=xyz$	$A=2(yz+xz+xy)$
Sphere: radius r	$V=\frac{4}{3}\pi r^3$	$A=4\pi r^2$
Right pyramid: height h dim x, y	$V=\frac{1}{3}hxy$	$A=xy+x\sqrt{(y/2)^2+h^2}+y\sqrt{(x/2)^2+h^2}$
Cylinder: height h radius r	$V=\pi hr^2$	$A=2\pi r(h+r)$
Right Cone: height h radius r	$V=\frac{\pi}{3}hr^2$	$A=\pi r\left(r+\sqrt{r^2+h^2}\right)$
Torus: radii $R>r$	$V=2\pi^2 r^2 R$	$A=4\pi^2 rR$
Tetrahedron: edge x	$V=\frac{1}{6\sqrt{2}}x^3$	$A=\sqrt{3}x^2$
Octahedron: edge x	$V=\frac{\sqrt{2}}{3}x^3$	$A=2\sqrt{3}x^2$
Dodecahedron: edge x	$V=\frac{15+7\sqrt{5}}{4}x^3$	$A=3\sqrt{20+10\sqrt{5}}x^2$
Icosahedron: edge x	$V=\frac{5\left(3+\sqrt{5}\right)}{12}x^3$	$A=5\sqrt{3}x^2$

1. (a) (3 points) What is the limit definition of the derivative of a function?

   (b) (6 points) Find dimensions of a rectangle with maximum area if one vertex is at the origin $(0, 0)$ and the opposite vertex on the curve $y = \frac{1}{1+x^2}$

   The maximum area rectangle has width ________________ and height ________________ .

2. (a) (3 points) Compute

$$\int x^{-2/5}\ dx$$

(b) (6 points) Let $g(x) = 2x^2 \log|x| - x^2$. Find the inflections points of f. Where is f concave up? Where is f concave down?

The inflection point(s) of g is (are) ____________________

g is concave up on ____________________

g is concave down on ____________________

3. (a) (3 points) Compute

$$\lim_{x\to\infty} \sqrt{\frac{25x^2}{7+4x^2}}$$

(b) (6 points) The function f is defined on positive numbers by $f(x) = x^3 - 3x + \frac{3}{x}$. The unique minimum of f can be approximated by Newton's Method. Give the iterating formula. Starting with initial guess $x_0 = 1$ find the first two iterations of the method. (You do not need to simplify the second iteration.)

The iterating formula is $x_{n+1} =$ ____________________.

$x_1 =$ ____________________.

$x_2 =$ ____________________.

4. (a) (3 points) Compute

$$\lim_{x\to\frac{1}{2}^+} \frac{d}{dx}\left|\frac{1}{x}-2\right|$$

(b) (6 points) A silo is to be built with a vertical cylindrical tube with no bottom but topped by a hemispherical dome. The silo must have a volume of 108π meters^3. The material for the hemispherical dome costs $\frac{8}{3}$ times the cost of the cylindrical tube material. Find the dimesions of the silo that minimize the cost.

The minimal cost silo has radius ______________ and height ______________ .

5. (a) (3 points) Suppose that f is a function with second derivative

$$f''(x) = (x-1)^4(x-2)^3(x-3)^2(x-4)$$

What are the inflection points of x?

(b) (6 points) Newton's Method can be used to approximate $\sqrt{15}$. Give the iterating formula for x_{n+1} in terms of x_n. Choose a reasonable initial guess x_0 and find the first iteration of the method using your guess.

The iterating formula is x_{n+1} =____________________.

Initial guess x_0 =____________________.

x_1 =____________________.

6. (a) (3 points) Find an antiderivative of

$$f(t) = \frac{-10}{(5t)^2 + 1}.$$

(b) (6 points) A spherical balloon is inflating at a rate of $3\text{m}^3/\text{min}$. How fast is the surface area increasing when the volume is $4\pi\text{m}^3$?

The surface area is increasing at the rate ____________________________.

7. (a) (3 points) Find all antiderivatives of

$$f(x) = 4\sec 3x \tan 3x$$

(b) (6 points) The table contains values of the function f and its derivative f' evaluated at different values of x.

x	$f(x)$	$f'(x)$
0	1	3
1	2	4
2	5	6

If f^{-1} is the inverse function and $g(x) = f^{-1}(f^{-1}(x))$, then what is $g'(5)$, the derivative of g evaluated at $x = 5$?

$g'(5) =$____________________.

8. (a) (3 points) Which of the following statement's about the limitations of Newton's Method are true? (Circle ALL that apply.)

   A. The method might not converge to any number.

   B. The method might converge to a number which is not a zero of the function.

   C. The method might converge to a zero of the function which is far away from the initial guess.

   (b) (6 points) Find the equation of the line tangent to the graph $y = \sin^2 x$ at the point $\left(\frac{2\pi}{3}, \frac{3}{4}\right)$.

9. BONUS (9 points): Multiple Choice

(a) The function $f(x)$ is continuous on $[a, b]$. Circle all of the following statements that must be true:

A. $f(x)$ is differentiable on (a, b).

B. There is a point c on $[a, b]$ where $f(c) = \frac{f(a)+f(b)}{2}$.

C. There is a point c on $[a, b]$ such that $f(c) \geq f(x)$ for all x on $[a, b]$.

D. There is a point c on (a, b) where $f'(c) = \frac{f(b)-f(a)}{b-a}$.

(b) The derivative of $\sin^{-1}(\cos[\ln(x^2)])$ is:

A. $\dfrac{2\sin(\ln[x^2])}{\sqrt{1-\cos^2[\ln(x^2)]}}$

B. $\dfrac{2\sin(2\ln|x|)}{(1+\cos^2[\ln(x^2)])}$

C. $\dfrac{-2\sin(2\ln|x|)}{x(1+\cos^2[\ln(x^2)])}$

D. $\dfrac{-2\sin(2\ln|x|)}{x\sqrt{1-\cos^2[\ln(x^2)]}}$

E. None of the above

(c) Consider the function $f(x) = \dfrac{(x-1)(x^2-4x+3)}{x-3}$ on the interval $[0, 3]$. Which of the following statements is true regarding the global/absolute minimum and maximum of f on this interval:

A. f has a global/absolute minimum at $x = 1/2$ and a global/absolute maximum at $x = 0$

B. f has a global/absolute minimum at $x = 1$ and no global/absolute maximum

C. f has a global/absolute minimum at $x = 1$ and a global/absolute maximum at $x = 3$

D. f has a global/absolute minimum at $x = 0$ and a global/absolute maximum at $x = 1$

E. None of the above.

(d) Which of the following best describes the behavior of $f(x) = x^2 + 3 - \sqrt{x^4 - 5}$ as $x \to \infty$:

A. f has a horizontal asymptote at $y = 0$

B. f has a horizontal asymptote at $y = 3$

C. $f \to \infty$ along the oblique asymptote $y = 2x$

D. $f \to \infty$

E. None of the above.

(e) Suppose $f(x)$, which is continuous for all x, has a global/absolute minimum at $x = 3$, and $f(3) < 0$. Further suppose that f has no maxima of any kind, and that $\lim_{x\to\infty} f(x) = \infty$ and $\lim_{x\to-\infty} f(x) = 0$. Circle all of the following statements that must be true regarding roots of f:

A. f has exactly one root

B. f has at least one root at a location $c > 3$

C. f has at least one root at a location $c < 3$

D. f has exactly one root on the interval $(-\infty, 3)$

E. None of the above.