**NAME:**

**Quiz 2:** No calculators. Justify all answers. No partial credit is given for an answer that is both unexplained and incorrect.

1. (2pts) Describe a reason why a limit might not exist.

   The left and right hand limits might give different values, the limit might diverge, or the function might have an essential singularity where there are multiple values arbitrarily close to the limt point.

2. (3pts) Compute the limit

$$\lim_{x\to 0} \frac{1-\cos x}{x^2}$$

   (Hint: Recall that $\lim_{x\to 0} \frac{\sin x}{x} = 1$ and the Pythagorean identity $\sin^2\theta = 1-\cos^2\theta$. )

   Multiply by a "tricky 1" to factor out some $\sin x/x$'s.

$$\begin{aligned}\frac{1-\cos x}{x^2} &= \frac{1-\cos x}{x^2}\frac{1+\cos x}{1+\cos x}\\ &= \frac{1-\cos^2 x}{x^2(1+\cos x)}\\ &= \frac{\sin^2 x}{x^2(1+\cos x)}\\ &= \frac{\sin x}{x}\frac{\sin x}{x}\frac{1}{1+\cos x}\end{aligned}$$

   Now we konw the limit value of each factor! And limits distribute over multiplication.

$$\begin{aligned}
\lim_{x\to 0}\frac{1-\cos x}{x^2} &= \lim_{x\to 0}\left(\frac{\sin x}{x}\frac{\sin x}{x}\frac{1}{1+\cos x}\right)\\
&= \left(\lim_{x\to 0}\frac{\sin x}{x}\right)\left(\lim_{x\to 0}\frac{\sin x}{x}\right)\left(\lim_{x\to 0}\frac{1}{1+\cos x}\right)\\
&= 1\cdot 1\cdot \frac{1}{2}\\
&= \frac{1}{2}
\end{aligned}$$