MATH 1551. Review problems for Midterm Exam 3.

This review problems are intended for you to practice. These problems might or might not be similar to the ones on the actual test. Studying this practice test alone will not prepare you for the test. Your best resources to prepare for the test: your book, class notes, homework, quizzes, worksheets.

1. Water is flowing at a rate of 50 m^3/min from a shallow concrete conical reservoir (vertex down) of base radius 45 m and height 6 m. (a) How fast (cm/min) is the water level falling when the water is 5 m deep? (b) How fast is the radius of the water's surface changing then?

2. A man walks along a straight path at a rate of 4 ft/sec. A searchlight is placed on the ground 20 ft from the path and is kept focused on the man. At what rate is the searchlight rotating when the man is 15 ft from the point on the path closest to the searchlight?

3. Two commercial airplanes are flying at an altitude of 40,000 ft along straight-line courses that intersect at right angles. Plane A is approching the intersection point at a speed of 442 knots (nautical miles per hour). Plane B is approaching the intersection at 481 knots. At what rate is the distance between the planes changing when A is 5 miles from the intersection point and B is 12 nautical miles from the intersection point?

4. Use differentials to estimate $(8.5)^{1/3}$.

5. Find the linearization of $f(x) = x + \frac{1}{x}$ at $x = 1$.

6. Find the linearization of $f(x) = \sqrt{1+x} + \sin x - .5$ at $x = 0$.

7. Estimate the volume of material in a cylindrical shell with length 30 in, radius 6 in, and shell thickness .5in.

8. How accurately should you measure the edge of a cube to be reasonably sure of calculating the cube's surface area with an error of no more than 2%?

9. Determine if the following statements are true or false. In each case, give a short explanation of your answer.

   (a) If f is continuous at $x = a$, then there exists the linearization of f around $x = a$: $L(x) = f(a) + f'(a)(x - a)$.

   (b) Use of df as an approximation of Δf is improved when dx is small.

   (c) The area of a triangle with sides of lengths a and b enclosing an angle θ is $A = (ab \sin\theta)/2$. If a and b are constant, then $\frac{dA}{dt} = 0$.

   (d) $\sin(a + dx) \approx \sin a + (\cos a)dx$

(e) $f(x) = |x^3 - 9x|$ has a local minimum at $x = 3$.

(f) If $f(x)$ is odd and f has a local maximum at $x = 3$, then $x = -3$ is also a local maximum.

(g) Every function has at least one critical point.

(h) The only local extreme value of $f(x) = (x-2)^{2/3}$ occurs at $x = 2$.

(i) If $f'' > 0$ on $[a, b]$, then f' has at most one zero in $[a, b]$.

(j) $f(x) = x - \ln x$ is decreasing on its domain.

(k) A quadratic curve $y = ax^2 + bx + c$ has at most one inflection point.

(l) A cubic curve $y = ax^3 + bx^2 + cx + d$ has at most one inflection point.

10. Show that the function $g(t) = \frac{1}{1-t} + \sqrt{1+t} - 3.1$ has exactly one zero in the interval $(-1, 1)$.

11. Verify that the function $f(x) = \ln(x-1)$ defined on $[2, 4]$ satisfies the hypothesis of the Mean Value Theorem, and find the value(s) of c for which $f'(c) = (f(b) - f(a))/(b - a)$.

12. Find the absolute and local extreme values of the function over its natural domain, and where they occur:

$$f(x) = \frac{x+1}{x^2 + 2x + 2}$$

13. Find the critical points and local and absolute extreme values for the function

$$f(x) = \begin{cases} -x^2/4 - x/2 + 15/4, & -55 \le x \le 1 \\ -x^3 - 6x^2 + 8x, & 1 < x \le 2 \end{cases}$$

14. Given that $f'(x) = (x-7)(x+1)(x+5)$, determine the critical points of f, the intervals where the function is increasing or decreasing, and points where f assumes local or absolute extreme values.

15. Find the open intervals where the function $f(t) = \frac{3}{2}t^4 - t^6$ is increasing or decreasing. Identify the local and absolute extreme values, if any, saying where they occur.

16. Sketch the graph of a differentiable function $y = f(x)$ through the point (1,1) if $f'(1) = 0$ and $f'(x) > 0$ for $x < 1$ and $f'(x) < 0$ for $x > 1$.

17. Find all critical points, the open intervals where the function is increasing and decreasing, the open intervals where the function is concave up or down, and identify local and absolute extreme values and inflection points for the functions

$$f(x) = \frac{1 - x^2}{2x + 1}, \qquad f(x) = x\sqrt{8 - x^2}, \qquad g(x) = x^{2/3}(x^2 - 4).$$

Use the information to sketch the graphs of $f(x)$ and $g(x)$.