

Math 1711-A
Summer 2016
Final Review
1 August 2016
Time Limit: 170 Minutes

Name: _____

1. How many subsets are there for the set of cards in a standard playing card deck? How many subsets are there of size 8?
2. What properties do the numbers p_1, p_2, p_3 need to have to be a probability distribution?
3. I toss a coin 7 times. How many outcomes have exactly 3 heads?
4. Rewrite the following system of equations as a matrix.

$$4x + 14y + 2z = 34$$

$$10y = 5x + 5y + 30$$

$$x = 3 + 8y + 11z$$

5. Suppose that the matrix from the previous problem has its reduced row echelon form below. What is the solution set to the system of equations?

$$\begin{pmatrix} 1 & 0 & -3 & 5 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

6. Consider the game with the payoff matrix shown below. Is the game determined? Find the optimal strategy for R . What is the value of the game?

$$\begin{pmatrix} 6 & 2 \\ 1 & 4 \end{pmatrix}$$

7. Consider the game with the payoff matrix shown below. Is the game determined? Find the optimal strategy for R . What is the value of the game?

$$\begin{pmatrix} 1 & -2 & 5 \\ 3 & 4 & 3 \end{pmatrix}$$

8. Consider the game with the payoff matrix shown below. Is the game determined? Find the optimal strategy for R . What is the value of the game?

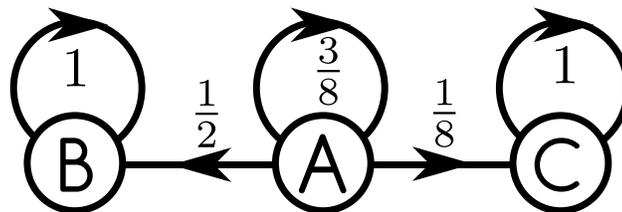
$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

9. Consider the system of equations shown below. How many solutions are there if $k = 14$?

$$x - y = 7$$

$$2x - 2y = k$$

10. What is the statistical definition of independent events?
11. Suppose that E and F are events with $P(E) = \frac{3}{10}$ and $P(F) = \frac{1}{5}$ and $P(E \cup F) = \frac{19}{50}$. Are E and F independent? Compute the following:
1. $P(E \cap F)$
 2. $P(E|F')$
 3. $P(E \cap F')$
 4. $P(F'|E)$
12. How many ways can a class of 24 students be divided into six study groups?
13. How many ways can a class of 24 students be assigned to Teams Red, Orange, Yellow, Green, Blue, and Purple if every team has 4 students?
14. I roll two 6 sided dice. What is the probability of rolling doubles or rolling sum 8?
15. A bag has 10 red marbles, 20 blue marbles, and 30 green marbles. If you draw 12 marbles, how many color arrangements can you obtain?
16. How many ways could you answer an exam of 5 multiple choice questions where each question has 4 choices?
17. I draw two cards from a deck. What is the probability of getting a pair of queens?
18. I roll a 20-sided die. Let E be the event $\{2, 4, 6\}$ and F be the event $\{1, 3, 5\}$. Are E and F independent? Why or why not?
19. Consider the Markov chain show below. Give the transition matrix in normal form. What is the stable matrix? Suppose that I start at state A. What is the probability I reach an absorbing state after 2 steps? What is the expected number of steps to get from state A to state C?



20. I toss a coin 5 times. What is the probability that I get heads every time given that the first 2 tosses were heads?
21. I roll a 6-sided die 6 times. What is the probability of obtaining exactly 3 twos?
22. Find the inverse of the matrix

$$\begin{pmatrix} 1 & -2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 5 \end{pmatrix}$$

23. A Family Feud survey asked 100 people "Name a pastime you can tell a person has just by looking at their shoes." Of the 30 who said "golf" 21 were over the age of 40 and 9 were under. Of the 70 who said something else, 57 were under the age of 40. What is the probability a person who said "golf" was under 40? What is the probability a person under 40 said "golf"? Are being under 40 and saying "golf" independent events? Why or why not?
24. Suppose Walter make a cheating deck of cards by replacing $5\heartsuit, 4\spadesuit,$ and $3\diamondsuit$ with three extra copies of $A\clubsuit$. How many times more likely is a flush in spades? If Walter deals two hands of 5 cards, how likely is he to deal more than one $A\clubsuit$ and get caught cheating?
25. Suppose 10% of a city is sick with the flu. After one week, 40% of the people who were sick are still sick and 30% of the people who were well got sick. What portion of the city is sick after two weeks? After many weeks, what portion of the city should we expect to be sick?
26. A basket has 5 red balls, 4 white balls, and 3 blue balls. If you draw with replacement, what is the probability of drawing two white balls in a row? Suppose you draw 10,000 balls, replacing each time. Use the approximation to the normal distribution to estimate the probability of getting between 2,000 and 3,000 blue balls. (Round to the nearest entry in standard normal table.)
27. Assume that 80% of all children who are exposed to chicken pox contract the disease. If 1225 children are exposed to chicken pox what is the probability that more than 945 of them will contract the disease? Approximate with the standard normal distribution.
28. A company is introducing a new soft drink and is planning to have 48 advertisements distributed among TV ads, radio ads, and newspaper ads. If the cost of a TV ad is \$500 each, the cost of a radio ad is \$ 200 each, and the cost of newspaper ad is \$200 each, how can the ads be distributed among the three types if the company has \$12,900 to spend on advertising?
29. Two fair dice are rolled. What is the probability that the numbers that appear add to 4?
30. What is the coefficient of x^4y^{10} in the expansion of $(x + y)^{14}$?
31. How many poker hands have 4 clubs and a card of another suit?
32. Use the matrix inverse shown below to solve the system of equations.

$$\begin{pmatrix} -5 & -2 & -2 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & 2 & 2 \\ 1 & 3 & 2 \\ 1 & 2 & 3 \end{pmatrix}$$

$$-5x - 2y - 2z = 1$$

$$-x + y = 2$$

$$-x + z = -3$$

33. 2000 lottery tickets are sold for on a new car worth \$7000. Each ticket costs \$5. If you buy one ticket, what is your expected winning?
34. Find the minimum of $3x + 2y$ subject to the constraints $x \geq 0$, $y \geq 0$, $2x + y \geq 6$, $x + y \geq 4$, $x + 2y \geq 5$.
35. How many 4 digit numbers end in 0, 1, or 2?
36. The transition matrix for a particular 3 state Markov chain is

$$A = \begin{pmatrix} 0.3 & 0.2 & 0.8 \\ 0.3 & 0.6 & 0.1 \\ 0.4 & 0.2 & 0.1 \end{pmatrix}.$$

Is A a regular stochastic matrix? Why or why not? Is A absorbing? Why or why not? What is the probability of moving from state 2 to state 3 in exactly 2 moves? Set up the system of equations to solve for the stable distribution of A .

37. Bob rolls a 6-sided die. If it's even, Bob must pay Fred that many dollars. If it's odd, Fred must pay Bob the number of dollars indicated by the number rolled. Find the expectation value of Fred's winnings.
38. A student is studying mathematics and chemistry. The probability that he passes mathematics is 0.75, the probability that he fails chemistry is 0.2, and the probability that he passes mathematics but fails chemistry is 0.05. Find the probability that he passes both courses.
39. A game has payoff matrix $\begin{pmatrix} 1 & -2 \\ -3 & 4 \end{pmatrix}$. Suppose that R uses the strategy with a 40% chance of move 1. C uses the strategy with a 20% chance of move 1. What is the expected value of the game?
40. A signal is made by placing 3 flags in order up a flag pole. If there are 10 different flags available, how many possible signals can be flown?
41. Five horses are running at a race track. You assume that every order of finish is equally likely. You bet that Son-of-a-Gun will win and that Gentle Lady will come in second. What is the probability that you will win both bets?
42. A pizza parlor offers onions, green peppers, pepperoni, olives and sausage as topping for the plain cheese base. How many different types of pizza can be made?
43. Experience with the Finite Mathematics course at a certain college suggests that of those students who get a grade of C or better on exam I, 90% get a C or better on exam II. Of those who get D or F on exam I, 70% get a D or F on exam II. This year 80% got C or better on exam I. Find the proportion of Finite Mathematics students who are likely to get D or F on exam II.

44. Find the stable distribution of the regular stochastic matrix

$$\begin{pmatrix} 0.6 & 0.3 \\ 0.4 & 0.7 \end{pmatrix}$$

45. Solve the system of equations.

$$2x - 6y = -63x + 2y = 13$$

46. Suppose that the probability that a federal income tax return contains an arithmetic error is 0.2. If 10 federal income tax returns are selected at random, find the probability that fewer than two of them contain an arithmetic error.
47. Among first-year students enrolled in a two-year program at a trade school, the following was determined to be true: 65% would successfully complete the year and go on to the second year; 18% would repeat the year; and 17% would drop out of the program. Among the second-year students, 82% would successfully complete the year and graduate; 66% would repeat the year; and 12% would drop out of the program. In the long run, what percentage of the second-year students will drop out of the program?
48. An investor has \$21,000 to invest in stocks, either conservative blue-chip or speculative stocks. If the stock market goes up, the speculative stocks could be worth \$56,000 while the blue chips would be worth \$37,000. If the market goes down, the blue chips would drop to \$14,000, while the speculative stocks could drop to \$11,000. Set up the payoff matrix. What is the optimal strategy and why?
49. How many poker hands have a pair and 3 aces?
50. A biased coin has a 40% rate of heads. If it is thrown twice, what is the chance it shows tails both times?
51. A Markov chain has the transition matrix

$$\begin{pmatrix} 1 & 0 & .2 & .1 \\ 0 & 1 & .1 & .1 \\ 0 & 0 & .5 & .1 \\ 0 & 0 & .2 & .7 \end{pmatrix}$$

Find the stable matrix. What is the expected number of moves a person starting in state 4 has to make before reaching an absorbing state? Suppose that 100 people start in state 3 and 50 people start in state 4. What is the probability that a person who eventually reaches state 1 started in state 3?

52. Consider the function $2x + 3y$ subject to the constraints $x \leq 5$, $y \leq 4$, $x + y \geq 4$, and $2x + 5y \geq 10$. Find the maximum. For what values of x and y does it occur? Find the minimum. For what values of x and y does it occur?