Ask yourself...

1. Do I realize that the homework questions are probably super relevant to the exam?

2. What is a set? What is a function? What is injective? What is surjective? What is bijective? What is cardinality? What are the subsets of a set and how many of them are there?

3. How many bijections are there between the twelve ribs on your left side and the months of the year? Prove by exhibiting an explicit bijection that $\mathbb{Z}$ and $\mathbb{Z}_+$ have the same cardinality. Give an example of a surjection $\mathbb{R} \to \mathbb{Z}$.

4. Most pianos can play 88 notes with 52 white keys and 36 black keys. The white keys are named A,B,C,D,E,F,G,A,B,C,...,E,F,G,A,B,C. How many sequences of 90 notes alternate between white keys and black keys? How many sequences of 49 notes play exactly 10 black keys, but never repeat a note? How many sequences of notes can I play so that the pitch always rises? How many sequences of 23 notes can I play so that the pitch never falls? How many sequences of 10 notes from the lowest 8 notes have exactly 3 As, 4Cs, and 2Es? Why does an octave only have seven notes? Don't pianos understand that oct means 8? If a *chord* is a combination of notes played at the same time, how many possible chords are there? How many chords consist of exactly 3 notes? How many possible songs can be made of 1 chord played for 1 sec every second for 60 seconds? What is the probability such a song *sounds good*, if all such songs are equally likely and any song sounds good if all its chords have at most 3 notes?

5. What is a binary string? How many length 10 binary strings are there? How many length 22 binary strings have their digits sum to 10 and the first 10 digits sum to 6? How many length 20 binary strings do not contain the substring 0101?

6. What is the Binomial Theorem? What is the Multinomial Theorem? What is a multi-choose?

7. What are anagrams, palindromes, and permutations? What is the probability that a random permutation of the positions of the letters in FIVECOMBINATORICSLEC-TURES will result in a word that contains the substring EVIL, if all permutations of the positions are equally likely?

8. What is recurrence and induction? What is the Well-Ordering Principle? How are inductive proofs structured and what are the important logical parts? Can I prove things like the following by induction or set up recurrence relations to count the following?

   (a) The Binomial Theorem

   (b) The number of tilings of a $2 \times n$ checkerboard by the Tetris tiles

   (c) If $a_0 = 2$ and $a_1 = 7$ and $a_{n+2} = 7a_{n+1} - 12a_n$ for any integer $n \geq 0$, then $a_n = 3^n + 4^n$.

   (d) $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$ for any positive integer n

(e) Every integer greater than 1 can be written uniquely as a product of a sequence of non-decreasing primes.

(f) If n pairwise-nonparallel lines are drawn in the plane so that no more than two every intersect in a single point, then they divide the plane into $\binom{n}{2} + \binom{n}{1} + \binom{n}{0}$ regions.

9. What is the pigeon hole principle? How can I use it to prove an upper bound on the minimal distance between a collection of points in a bounded geometry? E.g. Prove that if 17 points are placed in a sidelength 1 equilateral triangle, then there must be a pair of points at with 1/4 of each other. Can I use it to guarantee minimal amounts of structure? E.g. What's the fewest possible times any letters are repeated in a 200 length string of English letters?