Math 3012-A
Summer 2015
Exam 1
10 June 2015
Time Limit: 70 Minutes

Name: ______________________

This exam contains 8 pages (including this cover page) and 8 questions. There are 42 points in total. Any computable expression for a number is acceptable; there is no need to find a decimal representation. Write explanations or proofs clearly and in complete thoughts. Points are reserved for clarity. Do not include any scratch work that is not part of the proof in the proof. Use the blank side of paper for scratchwork. No calculators or notes may be used. Put your name on every page.

Grade Table

Question	Points	Score
1	6	
2	10	
3	2	
4	4	
5	6	
6	4	
7	6	
8	4	
Total:	42	

Formal Symbols Crib Sheet

$\neg$	not	$\wedge$	and	$\vee$	or
$\Rightarrow$	implies	$\lightning$	contradiction	$\in$	element of
$\forall$	for all	$\exists$	there exists	$\Leftrightarrow$	equivalence
$\varnothing$	empty set	$\mathbb{N}$	natural numbers	$\mathbb{Z}$	integers
$\mathbb{Z}_+$	positive integers	$\mathbb{Z}_{\geq 0}$	non-negative integers	$\equiv \pmod{n}$	congruence mod n
$\mathbb{Q}$	rationals	$\mathbb{R}$	reals	$\mathbb{C}$	complex numbers
$\times$	Cartesian product	$\subset$	subset	$\setminus$	set minus
$\cap$	intersection	$\cup$	union	$\mathcal{O}$	big-O asymptotic order
2^A	power set of set A	$\|A\|$	cardinality of set A	A^B	set of functions $B \to A$

1. (6 points) (a) Give the definition of a *bijection*.

(b) Give a bijection between the set of actors

$$A = \{Broderick, Crowe, Cumberbatch, Matthau, Redmayne, Weisz\}$$

and the set of mathematical innovators

$$M = \{Einstein, Feynman, Hawking, Hypatia, Nash, Turing\}.$$

Bonus if your bijection is *portrayed*.

(c) How many bijections are there between A and M?

2. (10 points) The set of (uncased) English letters is $\{A, B, C, \ldots, Z\}$ and has 26 elements.

   (a) How many 10 element subsets of English letters are there?

   (b) How many length 35 strings of English letters read the same forward and backward? (*Palindromes*)

   (c) How many distinct strings can one form by rearranging the letters of ROTORMOTORROOM?

   (d) How many five letter strings don't repeat a letter?

   (e) How many length 26 strings of English letters don't use every letter?

   (f) Bonus: If everyone has a first name of ten or fewer English letters how many people are required to guarantee that at least 3 people have the same name?

3. (2 points) Give a recurrence relation for the number of length n strings of English letters that do not contain the substring EVIL.

4. (4 points) Prove that for any connected graph with more than one vertex there must be a pair of vertices with the same degree.

5. (6 points) (a) Suppose that $G = (n, E)$ is a graph. Describe an algorithm that can validate if a function $f : n \to k$ is a k-coloring.

(b) What is the $\mathcal{O}$ complexity of your algorithm in terms of n?

(c) What complexity class are the following problems?

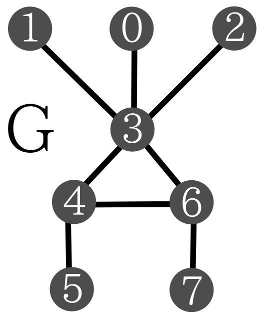
1. List all the permutations of a set size n. **P NP EXP**

2. Order a list size n. **P NP EXP**

3. Determine if a graph with n vertices is Hamiltonian. **P NP EXP**

4. Determine if a graph with n vertices is Eulerian **P NP EXP**

6. (4 points) Consider the connected graphs G and H described below.



$$H = (\{a, b, c, d, e, f, g, h\}, \{\{a, h\}, \{b, h\}, \{c, h\}, \{d, f\}, \{e, g\}, \{f, g\}, \{h, f\}, \{h, g\}\})$$

(a) Are G and H isomorphic? Give an isomorphism or explain why one cannot exist.

(b) Recall that the *symmetries* of G are the isomorphisms $G \to G$. How many symmetries does G have?

7. (6 points) A graph G has degree sequence 1,1,2,2,3,3,4,4,8

   (a) Can one determine if G has an Eulerian trail (not necessarily a circuit)? Explain how you know.

   (b) Can one determine if G is Hamiltonian? Explain how you know.

   (c) Can one determine if G is planar? Explain how you know.

8. (4 points) Prove that the chromatic number of a graph is at most one more than the maximal degree. (Hint: Induct on the maximal degree.)