The Twelvefold Way:
$|\{f : k \to n\}|$
How many ways to sort k balls into n boxes?

	Arbitrary any sorting	Injective max 1 ball per box	Surjective each box gets ball
Distinct Balls Distinct Boxes	n^k	$\frac{n!}{(n-k)!}$	$n!\left\{{k \atop n}\right\}$
Identical Balls Distinct Boxes	$\binom{n+k-1}{k}$	$\binom{n}{k}$	$\binom{k-1}{n-1}$
Distinct Balls Identical Boxes	$\sum_{j=0}^{n}\left\{{k \atop j}\right\}$	1 if $k \leq n$	$\left\{{k \atop n}\right\}$
Identical Balls Identical Boxes	$p_{\leq n}(k)$	1 if $k \leq n$	$p_n(k)$

$n!$ permutations of n

$\binom{n}{k}$ choose binomial coefficient

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

$$(1+x)^n = \sum_{k=0}^{n}\binom{n}{k}x^k$$

D_n derangements of n

$$D_n = !n = \text{round}\left(\frac{n!}{e}\right)$$

$\left\{{k \atop n}\right\}$ Stirling numbers of the 2nd kind

$$\left\{{k \atop n}\right\} = \frac{1}{n!}\sum_{j=0}^{n}(-1)^j\binom{n}{j}(n-j)^k = n\left\{{k-1 \atop n}\right\} + \left\{{k-1 \atop n-1}\right\}$$

B_n Bell numbers partitions of a set $B_n = \sum_{j=0}^{n}\left\{{n \atop j}\right\}$

$p_{\leq n}(k)$ integer partitions of k into at most n parts

$$\sum_{k=0}^{\infty} p_{\leq n}(k)x^k = \prod_{j=1}^{n}\frac{1}{1-x^j}$$

$p_n(k)$ integer partitions of k into exactly n parts

$$\sum_{k=0}^{\infty} p_n(k)x^k = \prod_{j=1}^{n}\frac{x}{1-x^j}$$

$$p_{\leq n}(k) = p_n(k+n)$$