Section 3.3 : Differentiation Rules

Chapter 3 : Differentiation

Math 1551, Differential Calculus

"A problem isn't finished just because you've found the right answer."

\- Yōko Ogawa

Section 3.3 Differentiation Rules

Topics

1. Derivative rules
2. Higher derivatives

Learning Objectives

For the topics in this section, students are expected to be able to:

1. Compute the derivative of a function using derivative rules.
2. Solve equations involving derivatives (for example, to locate points on a graph where the tangent line has a particular slope).

Derivative Rules

- Recall that we can compute derivatives using the limit definition

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

- There are rules that give us more efficient methods for computing derivatives of elementary functions.
- Proofs for most of derivative rules are in the textbook.
- For lecture, we will prove only one or two of the rules so that students have an understanding of where some of them come from.

Derivative Rules

Suppose $f(x)$ and $g(x)$ are differentiable functions, and $c \in \mathbb{R}$.

constant $$\frac{d}{dx}(c) =$$

sum rule $$\frac{d}{dx}(f(x) + g(x)) =$$

constant multiple $$\frac{d}{dx}\left(cf(x)\right) =$$

Derivative Rules

Suppose $f(x)$ and $g(x)$ are differentiable functions, and $n \in \mathbb{R}$.

power rule $\quad \frac{d}{dx}(x^n) =$

product rule $\quad \frac{d}{dx}\left(f(x)g(x)\right) =$

quotient rule $\quad \frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) =$

e^x $\quad \frac{d}{dx}\left(e^x\right) =$

Example 1

Determine the values of x that indicate where the slope of the tangent line of $y(x)$ is zero.

$$y(x) = \frac{x^2 + 12}{2x - 11}$$

Higher Derivatives

Suppose $f(x)$ is differentiable.

second derivative: $\frac{d}{dx}\left(\frac{d}{dx}f(x)\right) =$

third derivative: $\frac{d}{dx}\left(\frac{d}{dx}\left(\frac{d}{dx}f(x)\right)\right) =$

n^{th} derivative:

Example 2

Determine the values of t that indicate where the second derivative of the function is zero.

a) $y(t) = 3t^2 - 2t^3 + \dfrac{t^4}{2}, \quad t \geq 0$

b) $g(t) = e^t - \dfrac{t^2}{2}, \quad t \in \mathbb{R}$