Worksheet 10, Math 1551, Fall 2017

Sections from Thomas 13^{th} Edition: 4.3, 4.4: First and second derivative tests, curve Sketching.
Note: section 4.5 is not covered in this course. It is covered in Math 1552.

A Few Definitions and Theorems from Sections 4.1, 4.2, 4.3, 4.4

- **Local Extrema:** A function has a **local maximum** at $x = c$ if $f(x) \leq f(c)$ for all x in an open interval containing c. A function has a **local minimum** at $x = c$ if $f(x) \geq f(c)$ for all x in an open interval containing c.
- **Critical Points:** An interior point of the domain of $f(x)$ where $f' = 0$, or where f' is undefined, is a **critical point**.
- **MVT:** If $f(x)$ is a continuous function defined on $[a, b]$, and is differentiable over (a, b). Then there is at at least one point, $c \in (a, b)$, where

$$\frac{f(b) - f(a)}{b - a} = f'(c)$$

- **Increasing and Decreasing:** If $f'(x) > 0$ on (a, b), then f is **increasing** on $[a, b]$. If $f'(x) < 0$ on (a, b), then f is **decreasing** on $[a, b]$.
- **First Derivative Test:** Suppose f has a critical point at $x = c$.
  - If $f'(x)$ changes from positive to negative at c, then f has a **local maximum** at c.
  - If $f'(x)$ changes from negative to positive at c, then f has a **local minimum** at c.
  - If $f'(x)$ doesn't change sign from positive to negative at c, then f has no local minimum or maximum at c.
- The graph of a differentiable function $f(x)$ is
  - **concave up** on an open interval if $f''(x) > 0$
  - **concave down** on an open interval if $f''(x) < 0$
- An **inflection point** is a point where the graph of f changes concavity.
- **Second Derivative Test:** Suppose f has a critical point at $x = c$.
  - If $f''(c) > 0$, then f has a local minimum at c.
  - If $f''(c) < 0$, then f has a local maximum at c.
  - If $f''(c) = 0$, then the second derivative test is inconclusive.

Exercises

1. For each function below: (a) determine the interval(s) on which the function is increasing and/or decreasing; (b) Identify the local and absolute extreme values (if any) and where they occur.
   (a) $f(x) = \dfrac{x^3}{3x^2 + 1}$
   (b) $g(x) = x \ln x$
   (c) $h(x) = x^{2/3}(x + 5)$
2. If possible, sketch a curve or give a formula for a function that has the following properties. If it is not possible to do so, state why. Assume in each case that $f(x)$ is continuous, differentiable, and defined for all values of x.

(a) $f(x)$ has an inflection point at $x = 0$, and a critical point at $x = 0$.

(b) $g(x)$ is concave up on $[0, 4]$ and has a local maximum at $x = 2$.

(c) $h(x)$ is odd, has an inflection point at $x = 1$, is increasing on $[0, 2]$, is decreasing for $[2, \infty)$.

3. For $y(x) = \dfrac{x^2 - 4}{x^3}$, determine:

   (a) the domain

   (b) all asymptotes

   (c) symmetry (even, odd, neither)

   (d) locations of x and y intercepts (if any)

   (e) critical points, intervals where f is increasing/decreasing

   (f) inflection points and intervals of concavity

   (g) local and absolute extrema

   Use the information above to sketch $f(x)$. Label your axes.